Atomic Masses and Fundamental Constants 5

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INITIAL RESULTS FROM A NEW MEASUREMENT OF THE NEWTONIAN GRAVITATIONAL CONSTANT

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DESCRIPTION OF EXPERIMENT

The Newtonian gravitational constant, G, remains the least well-known of the fundamental constants of physics in spite of several recent and current efforts. One of these is underway at the National Bureau of Standards in cooperation with people from the University of Virginia. Our experiment is of the type originated by Beams et al.[1] In such an experiment gravitational force is balanced against the force of inertial reaction in an accelerated rotating reference frame. It is this acceleration which is measured to yield a value for G. In its present incarnation, we feel that the measurement has a potential accuracy of about 10 ppm. The principle of the measurement is recalled in Fig. 1. The small mass system (first a cylindrical bob and later a dumbbell), the large masses, the auto-collimator, the fiber which supports the small mass and an encoder disc are mounted on the rotating table. The auto-collimator senses the tendency of the small mass system to rotate toward alignment with a line joining centers of the large masses in response to the gravitational attraction. The torquer accelerates the rotating table so as to cancel the gravitational attraction, thus keeping the small mass stationary in the rotating frame. As seen in a laboratory-based frame the table accelerates uniformly over some (tolerable) number of revolutions. Clock readings at various table positions indicated by the disc encoder are the raw data of the measurement. The entire apparatus is enclosed in an acoustic chamber about 2.5 meters on an edge and mounted on a reinforced concrete slab of about 5000 kilograms.

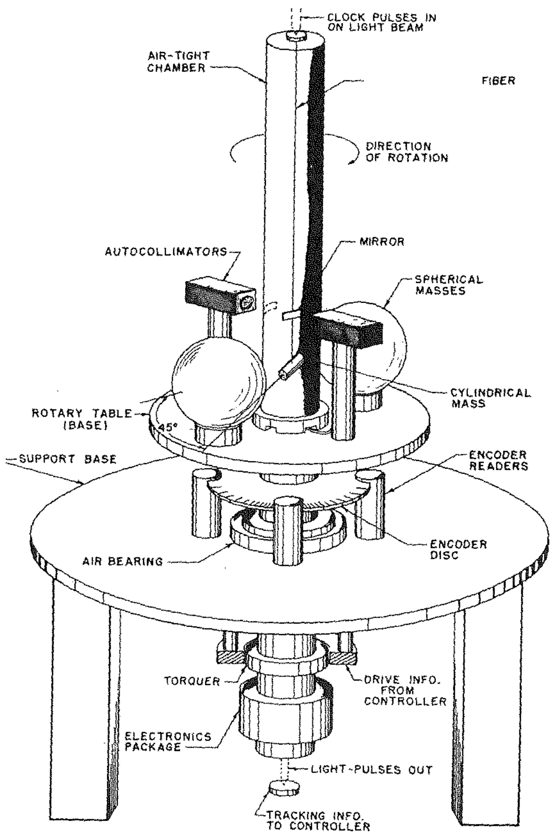

Fig. 1 Schematic of the Apparatus.

Accelerations are measured with the large masses in place and with the large masses removed. The difference between these two accelerations is due to the gravitational torque on the small mass introduced by the large masses. This acceleration may be converted to a value for G by means of Eq. [1].

$$G = \frac{R^3 I_T [\alpha_{on} - \alpha_{off}]}{Mm \quad K} \qquad \text{Eq. [1]}$$

where

R = 1/2 distance between the centers of large masses
I_T = Total moment of inertia of the small mass system
α = Acceleration of the table
M = Average mass of the large masses
m = Mass of the dumbbell
K = Geometric factors.

CONTRIBUTIONS TO THE ERROR

In order to measure G to 1 part in 10^5, the R in Eq. [1] must be known to 3 parts in 10^6. The uncertainty in the center of mass of the large masses limits the precision of the experiment to about 1 part in 10^5. These masses are the same ones used in the experiment of Beams et al.[1] They were machined from sintered tungsten by the Y-12 plant of Union Carbide. Their characteristics are listed in Table 1.

Table 1

Physical Characteristics of the Finished Tungsten Spheres

Size (inches)	Balance Period (sec)	Mass-Center Dislocation from the Geometrical Center (microinches)	Weight (kgs ± 0.00007)	Average Density (gms/cc)
4.001997	30	181.5	10.489980	19.073914
4.002011	24	298.0	10.490250	19.074866
4.000936	15	726.2	10.445982	19.009009

The time is recorded by counting 10 microsecond pulses from an NBS in-house frequency standard.

The positions of the large masses on the rotating table are determined by comparison with two sapphire rods epoxied to the rotating table, which are in turn compared to gauge blocks whose length determination has been carried out to better than 3 parts in 10^7.* The rotating table on which the large masses are mounted is a plate whose coefficient of thermal expansion is $1.5 \times 10^{-7}/°C$ and thus, the change in position of the large masses due to temperature is insignificant.

The auto-collimator signal which reflects the small mass system's angular position is converted to a digital signal by battery operated electronics mounted on the rotating system (there is no mechanical connection between the rotating system and the laboratory, all signals being transmitted to and from the rotating system via modulated light beams). A dedicated mini-computer converts this digital signal after amplification, integration and differentiation into an appropriate analog control signal as required to govern the torque.

*We are indebted to C. Tucker for this measurement and for valuable discussions.

The auto-collimator has a 2.5 cm aperture and consequently a diffraction pattern of a few seconds of arc. This diffraction pattern is divided into 10 bits (1024 parts) by the auto-collimator and electronics. The stability of the system is such that in operation the deviation of the small mass system is never more than a few bits with a standard deviation of about 1/3 of a bit under quiet conditions.

Twenty-four times per revolution as determined by the disc encoder, an elapsed time value accumulated from the house standard (as noted above) is stored in computer memory.

DATA REDUCTION

A run usually consists of 8 revolutions of the system (196 recordings of the time) with large tungsten masses on the table, 8 revolutions with the masses removed, and finally, 8 revolutions with the masses back on. Typically, the first revolution takes about half an hour and the 8th revolution is completed after about two hours. The times for each run are converted to an acceleration by a least-squares-fit to an equation of the form of Eq. [2].

$$\theta = C_\circ + C_1 t + C_2 t^2 + C_3 t^3 \qquad \text{Eq. [2]}$$

A typical output is shown in Fig. 2.

The meanings of the coefficients are as follows: $C_\circ$ = the initial position of the table in sectors (1 sector = $2\pi/24$ radians C_1 = the initial velocity of the table in sectors/second. C_2 = the acceleration of the table $\pi_\circ$/sectors/2/second^2. C_3 = the rate of change (instability) of the acceleration. The residuals shown in column 4, Fig. 2 are the residuals of the position, that is, the difference between the actual position of the table when the mark signal occurs and that position which it would have if the acceleration were uniform. Note that the residuals have a predominantly second-harmonic component, i.e., a period of 12 sectors and that their amplitude is inversely proportional to the number of revolutions, precisely what one would expect for a gravitational gradient horizontally across the room. The acceleration for the two "balls-on" runs are averaged and algebraically subtracted from the acceleration with the balls off to obtain the gravitational acceleration.

In our first measurement we used a small mass system whose weight was about 16 gm and whose moment of inertia was about 5 gm-cm^2. It was supported by a fused quartz fiber and had a torsion period of about 140 seconds. A table of the accelerations is given in Table 2.

BIG G 4 11 75 BALLS ON RUN 3 5.00

THIS IS TRY NUMBER 1 WITH N = 4 SET NUMBER 1 THE TOLERANCE FOR THIS FIT IS .0053

X	LAB.	CALC.	INT	RESID	WT
1878.23	21.0000	20.9989		.0011	1.00
1927.29	22.0000	21.9976		.0024	1.00
1974.93	23.0000	22.9972		.0028	1.00
2021.25	24.0000	23.9979		.0021	1.00
2066.37	25.0000	24.9993		.0007	1.00
2110.34	26.0000	26.0012		-.0012	1.00
2153.24	27.0000	27.0029		-.0029	1.00
2195.12	28.0000	28.0040		-.0040	1.00
2236.04	29.0000	29.0044		-.0044	1.00
2276.06	30.0000	30.0039		-.0039	1.00
2315.29	31.0000	31.0028		-.0028	1.00
2353.64	32.0000	32.0012		-.0012	1.00
2391.31	33.0000	32.9995		.0005	1.00
2428.32	34.0000	33.9982		.0018	1.00
2464.70	35.0000	34.9975		.0025	1.00
2500.49	36.0000	35.9974		.0026	1.00
2535.71	37.0000	36.9979		.0021	1.00
2570.39	38.0000	37.9988		.0012	1.00
2604.54	39.0000	38.9997		.0003	1.00
2638.17	40.0000	40.0004		-.0004	1.00
2671.31	41.0000	41.0008		-.0008	1.00
2703.96	42.0000	42.0008		-.0008	1.00
2736.16	43.0000	43.0004		-.0004	1.00
2767.92	44.0000	43.9997		.0003	1.00
2799.28	45.0000	44.9991		.0009	1.00
2830.24	46.0000	45.9986		.0014	1.00
2860.83	47.0000	46.9986		.0014	1.00
2891.07	48.0000	47.9990		.0010	1.00
2920.96	49.0000	48.9997		.0003	1.00
2950.51	50.0000	50.0005		-.0005	1.00
2979.73	51.0000	51.0013		-.0013	1.00
3008.63	52.0000	52.0019		-.0019	1.00
3037.20	53.0000	53.0020		-.0020	1.00
3065.46	54.0000	54.0017		-.0017	1.00
3093.41	55.0000	55.0011		-.0011	1.00
3121.08	56.0000	56.0003		-.0003	1.00
3148.48	57.0000	56.9993		.0007	1.00
3175.62	58.0000	57.9986		.0014	1.00
3202.50	59.0000	58.9982		.0018	1.00
3229.15	60.0000	59.9982		.0018	1.00
3255.56	61.0000	60.9985		.0015	1.00
3281.74	62.0000	61.9990		.0010	1.00
3307.69	63.0000	62.9996		.0004	1.00
3333.41	64.0000	64.0001		-.0001	1.00
3358.90	65.0000	65.0003		-.0003	1.00
3384.18	66.0000	66.0003		-.0003	1.00
3409.24	67.0000	67.0001		-.0001	1.00
3434.09	68.0000	67.9997		.0003	1.00
3458.75	69.0000	68.9994		.0006	1.00
3507.50	71.0000	70.9991		.0009	1.00
3531.60	72.0000	71.9994		.0006	1.00
3555.53	73.0000	72.9999		.0001	1.00
3579.28	74.0000	74.0005		-.0005	1.00
3602.86	75.0000	75.0010		-.0010	1.00
3626.26	76.0000	76.0013		-.0013	1.00
3649.50	77.0000	77.0014		-.0014	1.00
3672.56	78.0000	78.0012		-.0012	1.00
3695.96	79.0000	79.0007		-.0007	1.00

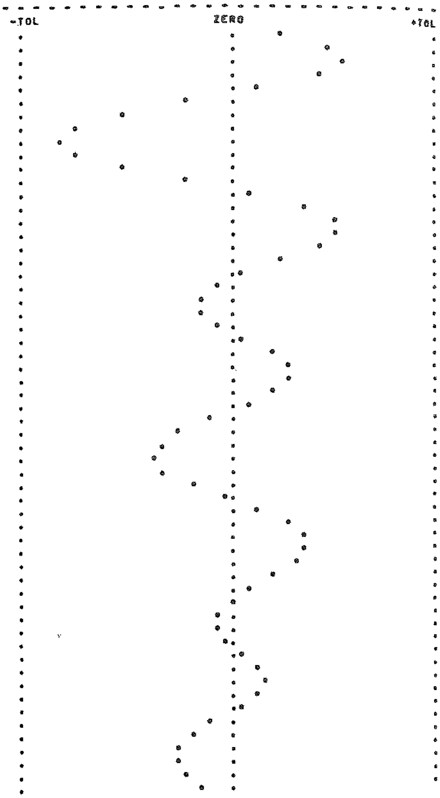

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Fig. 2. Computer Print-Out and Plot of a Typical Measurement.

Table 2

Acceleration Values Using the First Fiber and Small Mass System from Equation 2 with C_3 Set Equal to Zero

Date	Acceleration in Radians/Second^2
3-25-75	4.9684
3-27-75	4.9605
3-31-75	4.9486
4-01-75	4.9594
4-02-75	4.9608
4-09-75	4.9660
4-11-75	4.9476
Average	4.9586
	σ_m = .0042

Table 3

Acceleration Values Using the Second Fiber and Small Mass System from Equation 2 with C_3 Set Equal to Zero

Date	Acceleration in Radians/Second^2
5-06-75	5.69408
5-07-75	5.69414
5-08-75	5.68927
5-13-75	5.69649
5-14-75	5.69335
5-15-75	5.69419
Average	5.6936
	σ_m = .0012

The standard deviation of the mean of the results in Table 2 is about 9 parts in 10^4. Most of this error is believed to be caused by instabilities in the fiber's null position. A second small mass system was built with a mass of approximately 6gm and a moment of inertia the same as the original one: 7 gm-cm^2. This reduction in mass allowed a smaller fiber to be used whose torsion period is 360 seconds. The reduction in the mass while preserving moment of inertia was accomplished by making the bob in the shape of a dumbbell instead of a cylinder. The increase in the torsion period was due to the combined effect of the mass reduction and the use of extremely pure quartz for the torsion fiber. A typical set of accelerations for this new system are shown in Table 3.

The gravitational constant as calculated from the above acceleration is

$$6.6699 \times 10^{-11} \ \mathrm{N.m^2kg^{-2}}$$
$$\pm .0014$$

where the error includes only the statistical fluction in the acceleration. This value of G should be viewed with extreme care since it will be re-evaluated when the series of measurements with this fiber is finished and the metrology on the small mass system is completed.

The best fiber so far which supports the small mass system has a period of 1800 seconds which is expected to improve this result significantly. This fiber has not been installed to date.

Further work on this experiment is being planned using various small masses and other fibers, perhaps filamentary carbon fibers.

References

1. R.D. Rose, H.M. Parker, R.A. Lowry, A.R. Kuhlthau, and J.W. Beams, "Determination of the Gravitational Constant, G," Phys. Rev. Lett. 23, 655 22 Sept. 1969 .